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# B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

**BENGALURU – 560004**

**SEMESTER END EXAMINATION – SEPTEMBER- 2023**

**B.Sc. in Mathematics – 4th Semester**

**PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS**

**(NEP Scheme 2021-22 Onwards)**

**Course Code: MAT4DSC04** **QP Code: 4015**

**Time: 2 ½ Hours** **Max. Marks: 60**

***Instructions: Answer all the Sections.***

## SECTION-A

**I. Answer any SIX questions:** **(2X6=12)**

1. Solve $z = px + qy + \sqrt{\frac{pq}{p+q}}$.
2. Form the Partial differential equation for $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
3. Solve $\left(D^2 - 2DD' + D'^2\right)z = 0$.
4. Find the particular integral of $\left(2D^2 - DD' + 3D'^2\right)z = \sin(4x + y)$.
5. If $L\{f(t)\} = F(s)$, then prove that $L\{e^{at}f(t)\} = F(s - a)$.
6. Find the Laplace Transform of $tsint$.
7. Find $a_n$ for $f(x) = x + x^2$ in $(-\pi, \pi)$.
8. Obtain the Half range Sine series of $f(x) = x$ in $(0, \pi)$.

## SECTION-B

**II. Answer any FOUR questions:** **(4X6=24)**

1. Using Lagrange's multipliers solve pcotx + qcoty = cotz.
2. Solve $p^2x + q^2y = z$
3. Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by Charpit's method
4. Solve $\left(D^2 - 2DD' + D'^2\right)z = x + y$.
5. Find the solution for One Dimensional Wave equation using Fourier series.
6. Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the condition
   i) u(0, t)=0, u(1, t)=0 for all t,
   ii) u(x, 0)= $x^2 - x$ ; $0 \le x \le 1$.

## SECTION-C

**III. Answer any FOUR questions:** **(4X6=24)**

1. Find the Laplace transform for the following

i) $f(t) = \frac{\sin at}{t}$

ii) $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$.

2. Find the Inverse Laplace Transform of the following

i) $\frac{s+2}{s^2-2s+5}$ ii) $\log\left(\frac{s+a}{s+b}\right)$.

3. Verify convolution theorem for the functions $f(t) = e^t$ and $g(t) = cost$.

4. Obtain the Fourier series of the function $f(x) = x^2\ in\ (-\pi, \pi)$ , and hence deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{6}$.

5. Find the finite Fourier cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2$ in $(0, \pi)$.

6. Find the inverse finite Fourier sine and cosine transform of $\frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2}$ where $0 < x < 1$.

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